

IB HL Physics Internal Assessment

FINDING THE GAS CONSTANT OF AIR

Introduction

The most noteworthy lessons from my physics class were unit 3.2 (Modelling a Gas) and B.2 (Thermodynamics). It's exciting to predict the behaviour of an abstract entity as gasses, however, it disappointed me that all our equations were in terms of the behaviour of an *ideal gas*. Nothing in the real world is ever *ideal*, so how can those equations help me? This is why, in this paper, I'll be applying those ideal gas equations to ambient, real air.

Background

The main basis of this exploration will be the Ideal Gas Law:

$$pV = nRT$$

p = pressure of the gas (Pa)

V = volume of the gas (m^3)

n = number of moles of gas molecules (mol)

R = ideal gas constant ($Pa\ m^3\ mol^{-1}\ K^{-1}$)

T = Temperature of the gas (K)

This equation comes from the observations of Boyle's Law ($pV = constant$), Charles' Law ($\frac{V}{T} = constant$), and Guy-Lussac's Law ($\frac{p}{T} = constant$). It is, however, only applicable to an *ideal gas*. An ideal gas is a gas that obeys the following rules (Kinetic Theory of Gases | IB DP Physics HL FE2016, n.d.):

- The molecules in the gas are all monatomic and identical to each other.
- The particles occupy negligible volume compared to the volume of the gas.
- The particles only interact when they collide (there are no inter-particle forces).

- The particles experience fully elastic collisions with other particles and the walls of the container.
- The particles are moving around randomly following Newton's Laws of Motion.

Air is not an ideal gas. The molecules in air:

- Are primarily **not** monoatomic **not** all identical to each other.
 - 78.1% N₂, 20.9% O₂, etc. (Encyclopedia Britannica, 2023)
- The particles **do have** inter-particle forces.
- The particles **may not** experience fully elastic collisions with other particles and the walls of the container.

Research Question

Despite the air not being an ideal gas, I'll assume that air would behave in a similar way to the way the Ideal Gas Equation, Boyle's Law, Charles' Law, and Guy-Lussac's Law describes (at least in my operating range).

In this way, I can find the answer to my research question:

What is the Gas Constant of air?

Method

To begin, I took stock of my available materials in order to determine my independent and dependent variables. The independent variable was decided to be the volume (V) because I have access to a pneumatic cylinder, making the volume easy to manipulate just by compressing the piston. Out of the remaining variables (pressure, moles, and temperature), pressure (p) was determined to be the dependent variable

because it is hard to measure the number of moles of a gas at various states, and significantly varying the temperature may bring safety hazards and equipment damage. In this way, converting the ideal gas law into a linearized manner ($y = mx + b$) gives:

$$pV = nRT$$

$$p = \frac{nRT}{V}$$

$$p = R \left(\frac{nT}{V} \right)$$

From this equation, $\frac{nT}{V}$ would be graphed on the x-axis x allowing V to be the independent variable (since by convention, x is the independent variable, and both n and T can be held constant). Additionally, p will be graphed on the y-axis, allowing R to be the resulting gradient of the graphed line.

There is no way for me to directly know how many moles, n , of gas I will be using, but I can use air's ambient density, the volume of my pneumatic components, and the molar mass of air to determine the number of moles that begin in the experiment. As long as no air leaks out, the number of moles will remain constant.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{density} \times \text{volume} = \text{mass}$$

$$\frac{\text{mass} \times 10^3}{\text{molar mass}_{\text{grams}}} = n$$

$$\frac{\text{density} \times V_0 \times 10^3}{\text{molar mass}_g} = n$$

The density of air in my area (Houston, TX, USA) is, on average, $1.20317 \text{ kg m}^{-3}$ (Czernia & Szyk, 2024) and only varies slightly, especially in my experiment's timeframe. The V_0 in this equation is the uncompressed volume of my enclosed pneumatic system and is determined by adding up manufacturer-provided volumes or analysing CAD files

for all components of the circuit. This value for came to be $(4.62 \pm .12) \times 10^{-6} m^3$. Molar mass of air is taken to be $28.9647 g mol^{-1}$ (EngineeringToolbox.com, 2023).

To determine the temperature (T), I have used a thermometer to notice that a corner of my home that is relatively well-isolated from fans, air-conditioning, and outdoor temperature cycles is usually at $63.5^\circ F = 17.5^\circ C$. I will conduct the experiment in this area in order to keep my temperature as constant as possible.

All the relevant variables:

p = dependent variable

V = independent variable

$n = (1.92 \pm) \times 10^{-4} mol$

$T = 290.5 \pm .6 K$

Procedure

To collect data the following materials were used:

- Air Cylinder
- Pressure Guage
- Pneumatic Tubing
- Teflon Tape
- Pneumatic inserts
- Digital Callipers
- Inelastic String
- Isolated, constant temperature location

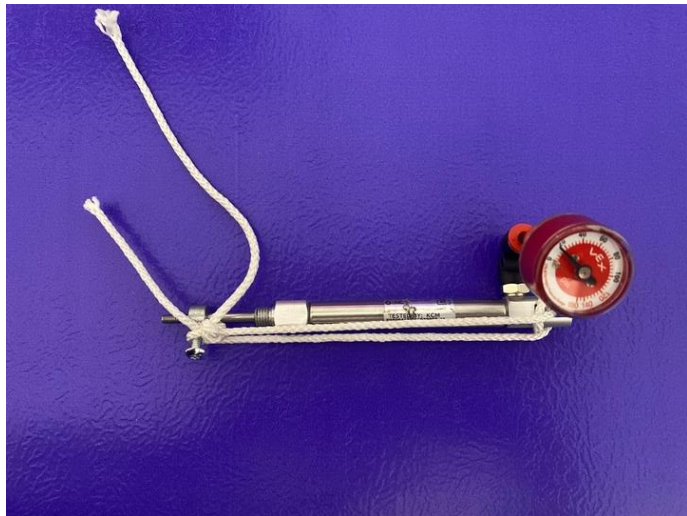
Here were the steps to the experiment:

1. Create the following pneumatic circuit configuration. Use Teflon Tape, proper fittings, and correct tubing to ensure the circuit is leak-proof.



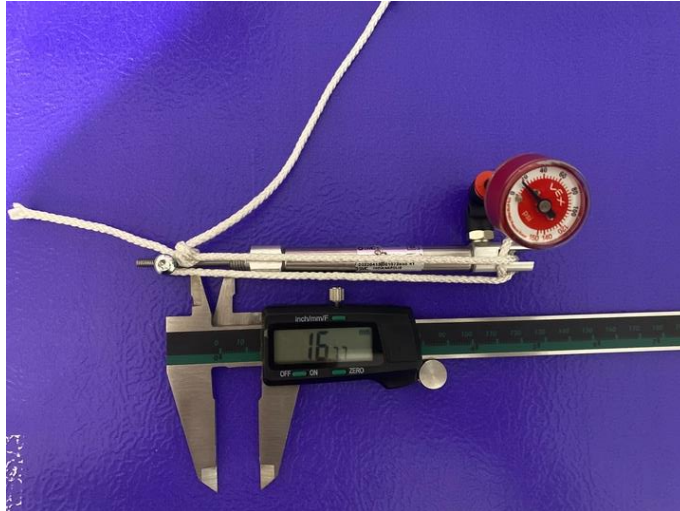
a.

2. Retract the cylinder bore by tying the string to restrict the volume.



a.

3. Let the system sit for some time (at least 1 hour) in an isolated location for the air in the cylinder to come to equilibrium with the ambient temperature.
4. Measure and note the distance from the ending of the housing to the tip of the bore (for measuring volume):



a.

5. Note the reading from the pressure gauge.
6. Repeat steps 2-5 for more data points.

Data

Using the lab's method (as explained above), this was the raw data:

P (PSI)	ΔP (PSI)	L (mm)	ΔL (mm)
0	1	53.03	0.01
8.5	1	26.99	0.01
14	1	20.48	0.01
20	1	14.4	0.01
12	1	22.24	0.01
5.5	1	29.83	0.01
27.5	1	10.64	0.01
30	1	8.94	0.01
9	1	24.44	0.01
32	1	7.58	0.01
39.5	1	5.72	0.01
46	1	2.33	0.01

The uncertainty in pressure was determined through self-analysis to be ± 1 PSI. The uncertainty of L was taken as ± 1 of the last digit in the instrument (digital calipers). Now, these base values were processed further:

Pressure-Pa (Pa)	ΔP (Pa)	Volume (m ³)	ΔV (m ³)
101352.932		4.610E-006	8.694E-010
159958.369	18818.632	2.565E-006	9.503E-010
197879.534	14134.252	2.054E-006	1.003E-009
239248.078	11962.404	1.576E-006	1.095E-009
184090.020	15340.835	2.192E-006	9.856E-010
139274.097	25322.563	2.788E-006	9.346E-010
290958.758	10580.318	1.281E-006	1.204E-009
308195.651	10273.188	1.147E-006	1.283E-009
163405.748	18156.194	2.365E-006	9.676E-010
321985.165	10062.036	1.041E-006	1.373E-009
373695.845	9460.654	8.944E-007	1.564E-009
418511.768	9098.082	6.282E-007	2.696E-009

Note that the units have been converted to all SI units. Also, the first row of data that was 0 PSI from the gauge is now non-zero because the ambient air pressure was added to the gauge's relative measurements. Here is how the spreadsheet calculates Pressure (Pa):

$$p_{Pa} = 6894.75729 \cdot (p_{PSI} + 14.7)$$

Where 6894.75729 is the conversion factor from PSI to Pascals, and 14.7 is my ambient air-pressure (in PSI).

This is the formula for volume, derived from the volume of a cylinder ($V = h\pi r^2$):

$$V = ((L - 2.33) \times 10^{-3} \cdot \pi \cdot (5 \times 10^{-3})^2) + 6.28196 \times 10^{-7}$$

In this formula, 2.33×10^{-3} is the constant difference between the bore length (L) and the cylinder of gas' height. 5×10^{-3} is the radius of the piston in the cylinder. Both these values will vary depending on the model of cylinder used. The addition of 6.28196×10^{-7} accounts for the volume in the pneumatic fitting, tube, and pressure gauge. This value will also be different depending on each system's setup.

Afterwards, more calculations were done to obtain the final graphing values:

1/Volume (m ⁻³)	Δ 1/v (m ⁻³)	Moles (mol)	Δn (mol)	T (K)	ΔT (K)	1/V n T (m ⁻³ mol K)	Δ 1/V n T (m ⁻³ mol K)
216911.991	40.904	1.92E-004	2.001E-005	290.5	0.6	2085E+004	1288.323
389865.391	144.448	1.92E-004	2.001E-005	290.5	0.6	.1722E+004	2319.511
486927.538	237.758	1.92E-004	2.001E-005	290.5	0.6	.7129E+004	2900.179
634448.692	440.589	1.92E-004	2.001E-005	290.5	0.6	.5349E+004	3786.114
456220.251	205.135	1.92E-004	2.001E-005	290.5	0.6	.5419E+004	2716.302
358674.789	120.240	1.92E-004	2.001E-005	290.5	0.6	.9984E+004	2133.237
780724.308	733.763	1.92E-004	2.001E-005	290.5	0.6	.3499E+004	4669.698
871578.043	974.920	1.92E-004	2.001E-005	290.5	0.6	.8560E+004	5221.795
422884.596	173.030	1.92E-004	2.001E-005	290.5	0.6	.3561E+004	2516.870
961048.663	1267.874	1.92E-004	2.001E-005	290.5	0.6	.3545E+004	5768.577
1.118E+006	1954.564	1.92E-004	2.001E-005	290.5	0.6	.2291E+004	6737.444
1.592E+006	6832.017	1.92E-004	2.001E-005	290.5	0.6	.8691E+004	9818.587

All figures are being displayed and processed to the maximum amount of significant figures possible to avoid any over-rounding. Each value has an accompanying uncertainty determined by uncertainty propagation rules. For example:

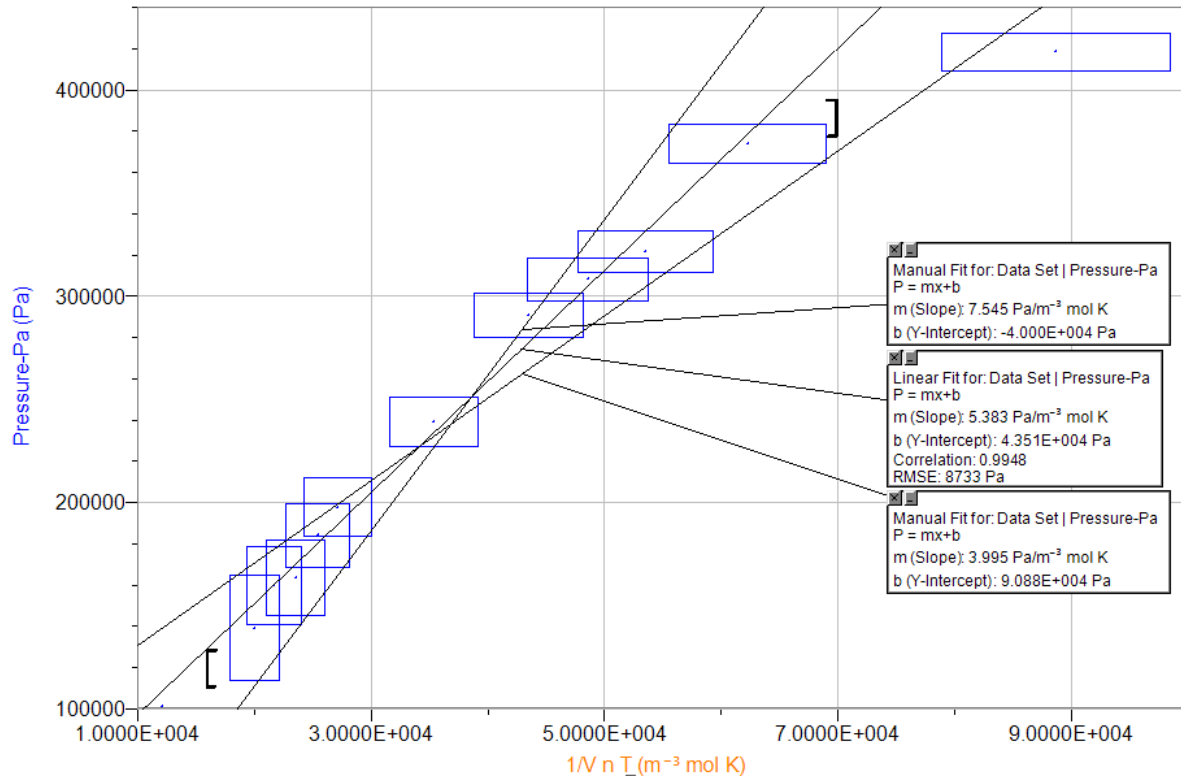
$$\Delta \frac{1}{V} nT = \frac{1}{V} nT \times \left(\frac{\Delta \frac{1}{V}}{\frac{1}{V}} + \frac{\Delta n}{n} + \frac{\Delta T}{T} \right)$$

Analysis

This was all the initial data:

P (PSI)	ΔP (PSI)	L (mm)	ΔL (mm)	Pressure-Pa (Pa)	ΔP (Pa)	Volume (m ³)	ΔV (m ³)	1/Volume (m ⁻³)	Δ 1/v (m ⁻³)	Moles (mol)	Δn (mol)	T (K)	ΔT (K)	1/V n T (m ⁻³ mol K)	Δ 1/V n T (m ⁻³ mol K)
0	1	53.03	0.01	101352.932		4.610E-006	8.694E-010	216911.991	40.904	1.92E-004	2.001E-005	290.5	0.6	2085E+004	1288.323
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With all these values, the data (including error bars) was graphed in a linearized way to find the value of R . Manual minimum and maximum best fit lines were added to the graph too:



From the graph, the value of R is $5.4 \pm 1.8 \text{ Pa m}^3 \text{ mol}^{-1} \text{ K}^{-1}$.

Note that the first and last data points were not used in the calculation of this value (only the points within the black brackets were considered). This is because both those points are behavioural outliers. The lowest point (only visible as a small blue point in the bottom-left) reading 0 PSI on the gauge is likely tainted by the fact that it's low-pressure differential may not have been able to overcome the slight frictions within the mechanism of the pressure gauge. The highest point was recorded at the full pressing of the piston, leaving no room within the gas cylinder thereby introducing new volume and temperature dynamics which would be inconsistent with all the other points. Some of these confounding dynamics could include: the whole section of gas being cut into two

parts from the piston's full retraction, gas occupying space around the piston, or the gas seeping into 'retractive' section of the pneumatic cylinder.

Conclusion

All in all, this exploration has led me to find the Gas Constant of air to be $5.4 \pm 1.8 \text{ Pa m}^3 \text{ mol}^{-1} \text{ K}^{-1}$. This value for R is inconsistent with the accepted ideal gas constant of $8.3145 \text{ Pa m}^3 \text{ mol}^{-1} \text{ K}^{-1}$ (LibreTexts, 2023). These values have a percent error of about 35%.

This is especially unexpected because according to Helmenstine (2022), air should behave very similarly to an ideal gas under "near room temperature and atmospheric pressure". Although this experiment did pressurize the air in the pneumatic system, the compression factor charts for "various gasses at 273K" (Non-ideal Behaviour of Gases (Article) | Khan Academy, n.d.) seem to suggest that this deviance of air from ideal gas behaviour happens only after a significant pressure of about 40 *Bar* has been reached. This is not near my maximum pressure of about 4 *Bar*.

One reason this experiment may not have yielded an R value closer to 8.3 is because the gas was constantly restricted to a low volume, $2.56 \times 10^{-6} \text{ m}^3$ at maximum. In low-volume scenarios, intermolecular forces would systemically change pressure readings. Low volume brings the molecules of gas closer to each other, thereby increasing the strength of any electrical forces between molecules, violating one of the foundational rules for an ideal gas that does not account for intermolecular forces. From this increase in intermolecular forces that comes with a smaller volume, energy that would contribute to a pressure reading gets lost to the work done by those forces. In this way, the overall pressure readings of the system are diminished. This hypothesis

is consistent with the observation that if pressure was overall measured to be higher, the experimental value for R would also be higher, bringing it closer to 8.3145.

Another reason for this experimental value of R to be lower than the expected value is because of the quality and purpose of my pressure gauge. The pressure gauge I used is primarily meant to be used as a quick-reference component of robot pneumatic circuits (Vex Robotics, 2024). Firstly, this gauge is not a precision instrument, it is meant to be used as an approximate reference for robot-users. This can also be seen in the gauge's unconventional 4 PSI per tick mark scaling. Secondly, the gauge is meant to be mainly used in higher pressure systems. The robots that this product is meant for operate at a relative pressure of about 100 *PSI*. This significant difference between my maximum relative pressure of 39.5 *PSI* and the intended operating pressure may have contributed to a systematically lower reading of pressure values since the manufacturer likely expects the robots' higher operating pressures to make the frictions within the mechanisms more negligible. In this case, the frictions in the gauge would contribute to a notably lower pressure reading because the pressure of the air is not high enough to negate the resistance in the gauge. I was able to notice the combination of these two effects during the experiment where if I did not change the bore-length substantially enough, the pressure would appear to read the same despite the change in volume.

The constants and independent variables likely had far fewer chances of contributing to the 35% difference between this experiment's R and an ideal gas' R .

Firstly, temperature (T) was kept constant by performing the experiment in a location relatively isolated from outdoor, air-conditioning, or weather-related temperature changes. Additionally, the whole system was only measured after 1-2 hours after each volume change to allow the gas to come in equilibrium with the relatively constant ambient temperature. In addition to this, even decreasing the temperature by 10 *Kelvin* (a completely unrealistic variance or measurement error in this setup) would only increase the experimental value of R to about 5.6, further supporting the claim that the temperature-term was not the source of error.

Secondly comes the constant for moles (n). This value was derived by performing calculations from air's ambient density, the molar mass of air, and the volume of my pneumatic components. Air's ambient density and molar mass were both values from trusted sources, so the main suspect of error would be in my self-derived volume calculations. Any errors in the volume calculations would cause a systematic error not only in the amount of moles considered, but also in the dependent variable's volume calculations. However, fault in volume determination is unlikely because all the calculations were based on manufacturer specifications, measurements from CAD files, and measurement using calibrated digital callipers. In this way, it is doubtful that either the number of moles or the dependent variable, volume, had a systemic error giving rise to a lower R value.

Overall, although the end-result of this experiment is inaccurate to the expected results, the methodology, calculations, and theory behind this exploration are still strong. This experiment could be repeated with a larger pneumatic cylinder and a more accurate pressure gauge as a retry to determine a more accurate gas constant for air. Doing this would better determine whether air truly does behave similarly to an ideal gas under these conditions or whether air does have a separate gas constant even under 'normal' conditions. Additionally, this experiment can be performed on various other gasses to determine their empirical gas constants too.

This experiment's final graph did show a linear relationship between p and $\frac{nt}{V}$, but if it exceeded the bounds of air's ideal-gas behaviour, the graph would have become non-linear. Therefore, this experiment could also be used as a test for the approximate range of any gas' behaviour to be predicted with its gas constant.

To further extend the experiment, one could also use the Van Der Waals equation $[(p + \frac{an^2}{V^2})(V - nb) = nRT]$ as the basis of their experiment instead of the Ideal Gas Law equation. In this way, instead of determining R one would either be determining the Van Der Waals constants of a and b or using all accepted constants to test this equation's empirical accuracy and application ranges for different gasses.

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